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Dynamical generation of four-dimensional space-time in the IIB matrix model

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Abstract: We study the spontaneous breakdown of SO(10) symmetry in the IIB matrix model, a conjectured nonperturbative definition of type IIB superstring theory in ten dimensions. Our analysis is based on a gaussian expansion technique, which was originally proposed by Kabat-Lifschytz and applied successfully to the strong coupling dynamics of the Matrix Theory. We propose a prescription for including higher order corrections, which yields a rapid convergence in a simple example. This prescription is then applied to the IIB matrix model up to the third order. We find that the 'self-consistency equations' allow various symmetry breaking solutions. Among them, the solution preserving SO(4) symmetry is found to have the smallest free energy. The value of the free energy comes closer to the analytic formula obtained by Krauth-Nicolai-Staudacher as we increase the order. The extent of the space-time in the 4 directions is larger than the remaining 6 directions, and the ratio increases with the order. These results provide the first analytical evidence that four-dimensional space-time is generated dynamically in the IIB matrix model.

Keywords: Matrix Models, Superstring Vacua, Superstrings and Heterotic Strings.

Contents

1. Introduction

One of the recent excitements in string theory is the appearance of concrete proposals for its nonperturbative definitions. The IIB matrix model [1], which was conjectured to describe type IIB superstrings in 10 dimensions, has a particularly simple form. It is a supersymmetric matrix model, which can be obtained from the zero-volume limit of 10d SU(N) super Yang-Mills theory. The space-time is represented by 10 bosonic matrices, and hence treated dynamically. This allows us in particular to investigate the possibility [2] that our 4d space-time is generated *dynamically* in superstring theory in 10d. Since the model has manifest SO(10) symmetry, the emergence of 4d space-time requires the SO(10) symmetry to be spontaneously broken. However, the absence of SSB has been concluded in various simplified versions of the IIB matrix model [3, 4, 5]. (See also ref. [6].) These results suggested that the *phase* of the fermion integral must play a crucial role if the SSB really takes place. A saddle-point analysis of its effect [7] led to the conclusion that the SO(10) symmetry can be spontaneously broken down to SO(d), where $3 \leq d \leq 8$. Monte Carlo studies have been attempted in ref. [8] and an intuitive argument for $d = 4$ has been presented. Ref. [9], on the other hand, has provided concrete examples of exactly solvable matrix models, in which SO(D) symmetry is spontaneously broken precisely due to the phase.

In this paper, we address the issue of SSB in the IIB matrix model by using the gaussian expansion technique developed in refs. [10, 11]. Such a method for general supersymmetric models was originally proposed by Kabat-Lifschytz [12] with the particular intention of studying the strong-coupling regime of the Matrix Theory [13]. Even at the *leading order* of the expansion, the result turned out to be consistent with the conjectured duality to supergravity, and various nonperturbative blackhole dynamics have been extracted successfully [14]. However, the possibility of a systematic improvement beyond the leading order was left unclear. Here we will propose a prescription for including higher order corrections systematically, which indeed yields a rapid convergence to the exact result in a simple example.

We apply this prescription to the IIB matrix model and carry out calculations up to the third order. 'Self-consistency equations' allow various solutions which preserve only some subgroup of SO(10). Among them, the solution preserving SO(4) symmetry gives the smallest free energy. The value of the free energy comes much closer to the analytic formula obtained by Krauth-Nicolai-Staudacher [15] as we increase the order. As a fundamental observable which probes the space-time structure, we calculate the extent of space-time in each direction. For the SO(4) preserving solution, the extent in the four directions is found to be larger than the remaining six directions and moreover the ratio becomes larger as we go to higher order. These results provide the first analytical evidence that four-dimensional space-time is generated dynamically in the IIB matrix model.

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2. Bosonic Yang-Mills Integral

In order to illustrate our method, let us consider the bosonic Yang-Mills integral defined by

$$Z = \int \mathrm{d}A \, \mathrm{e}^{-S} \,, \tag{2.1}$$

$$S = -\frac{1}{4} N\beta \ \mathrm{tr}[A_\mu, A_\nu]^2 \,, \qquad \beta = \frac{1}{g^2 N} \,. \tag{2.2}$$

The bosonic matrices A_μ ($\mu = 1, \ldots, D$) are $N \times N$ hermitean matrices, which we expand as $A_\mu = A_\mu^a T^a$ with respect to the SU(N) generators T^a ($a = 1, \ldots, (N^2 - 1)$) normalized as $\mathrm{tr}(T^a T^b) = \frac{1}{2}\delta^{ab}$. The integration measure for A_μ is defined as $\mathrm{d}A = \prod_{a=1}^{N^2-1} \prod_{\mu=1}^{D} dA_\mu^a/\sqrt{2\pi}$. As an important consequence of the zero-volume limit, one can actually absorb the parameter g by rescaling the dynamical variables $A_\mu \mapsto \sqrt{g} A_\mu$. Therefore, the parameter g is merely a scale parameter rather than a coupling constant. The partition function is conjectured to be finite [16] for $N > D/(D-2)$, and this conjecture was proved in [17]. A systematic $1/D$ expansion has been formulated in [3]. In particular the absence of SO(D) breaking is shown to all orders of the $1/D$ expansion and this conclusion is confirmed by Monte Carlo simulations [3] for various $D = 3, 4, 6, \ldots, 20$. The model has been studied by the gaussian expansion assuming that the SO(D) symmetry is not spontaneously broken [10], and the numerical results of the VEVs of Polyakov line and Wilson loop [18] have been reproduced qualitatively.

In order to examine the SSB of SO(D) symmetry by means of the gaussian expansion, we repeat the calculation of ref. [10] without assuming the absence of SO(D) breaking. It turns out to be convenient to introduce the rescaled dynamical variables X_μ given by $X_\mu = \beta^{1/4} A_\mu$, so that the action takes the canonical form

$$S = -\frac{1}{4} N \, \mathrm{tr}[X_\mu, X_\nu]^2 \,. \tag{2.3}$$

The most general SU(N) invariant gaussian action without assuming SO(D) symmetry can be brought into the form

$$S_0 = \sum_{\mu=1}^{D} \frac{N}{v_\mu} \operatorname{tr}(X_\mu X_\mu) , \qquad v_\mu > 0 , \tag{2.4}$$

by making an appropriate SO(D) transformation. Then we rewrite the partition function (2.1) as

$$Z = Z_0 \, \langle \mathrm{e}^{-(S-S_0)} \rangle_0 , \tag{2.5}$$

$$Z_0 = \int \mathrm{d}A \, \mathrm{e}^{-S_0} = \beta^{-D(N^2-1)/4} \int \mathrm{d}X \, \mathrm{e}^{-S_0} , \tag{2.6}$$

where $\langle \, \cdot \, \rangle_0$ is a VEV with respect to the partition function Z_0. From (2.5) it follows that the free energy $F = -\ln Z$ can be expanded as

$$F = \sum_{k=0}^{\infty} F_k ; \qquad F_0 = -\ln Z_0 ,$$

$$F_k = -\frac{(-1)^k}{k!} \langle (S - S_0)^k \rangle_{\mathrm{C},0} \qquad (\text{for } k \geq 1) , \tag{2.7}$$

where the suffix 'C' in $\langle \, \cdot \, \rangle_{\mathrm{C},0}$ means that the connected part is taken. The first two terms of the expansion are given as

$$F_0 = \frac{1}{2}(N^2-1) \left\{ D \ln(N\beta^{1/2}) - \sum_{\mu=1}^{D} \ln v_\mu \right\} , \tag{2.8}$$

$$F_1 = \langle S \rangle_0 - \langle S_0 \rangle_0 , \tag{2.9}$$

$$\langle S \rangle_0 = \frac{1}{4}(N^2-1) \sum_{\mu \neq \nu} v_\mu v_\nu , \tag{2.10}$$

$$\langle S_0 \rangle_0 = \frac{1}{2}(N^2-1) D . \tag{2.11}$$

The variational parameters v_μ in the gaussian action (2.4) can be determined in such a way that the free energy F calculated up to the first order becomes minimum. This gives the self-consistency equations

$$0 = \frac{1}{N^2-1} \frac{\partial}{\partial v_\mu} (F_0 + F_1) = -\frac{1}{2 v_\mu} + \frac{1}{4} \sum_{\nu \neq \mu} v_\nu . \tag{2.12}$$

Considering $v_\mu > 0$, one immediately finds that

$$v_1 = v_2 = \cdots = v_D = \sqrt{\frac{2}{D-1}} , \tag{2.13}$$

which agrees with ref. [10]. Thus the gaussian approximation is able to reproduce correctly the absence of the SO(D) symmetry breaking.

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3. The IIB matrix model

Let us move on to the IIB matrix model, which is defined by the partition function

$$Z = \int \mathrm{d}A\mathrm{d}\Psi \mathrm{e}^{-S}\,; \qquad S = S^{(\mathrm{B})} + S^{(\mathrm{F})}\,, \tag{3.1}$$

where $S^{(\mathrm{B})}$ is the bosonic action given by (2.2) with $D = 10$, and $S^{(\mathrm{F})}$ is given by

$$S^{(\mathrm{F})} = -\frac{i}{2} N\beta \,\mathrm{tr}\Big(\Psi_\alpha (\widetilde{\Gamma}_\mu)_{\alpha\beta} [A_\mu, \Psi_\beta]\Big)\,. \tag{3.2}$$

The fermionic matrices Ψ_α ($\alpha = 1, \ldots, 16$) are traceless $N \times N$ hermitean matrices, which we expand as $\Psi_\alpha = \Psi_\alpha^a\, T^a$. The integration measure is given by $\mathrm{d}\Psi = \prod_{a=1}^{N^2-1} (\prod_{\alpha=1}^{16} \mathrm{d}\Psi_\alpha^a)$. The 16×16 symmetric matrices $\widetilde{\Gamma}_\mu$ are given as $\widetilde{\Gamma}_\mu = \mathcal{C}\,\Gamma_\mu$, where Γ_μ are the 10d gamma matrices after Weyl projection, and $\mathcal{C}$ is the charge conjugation matrix, which satisfies $\Gamma_\mu^\top = \mathcal{C}\Gamma_\mu \mathcal{C}^\dagger$ and $\mathcal{C}^\top = \mathcal{C}$. The partition function (3.1) is conjectured [15] to be finite for arbitrary N, and this conjecture was proved in [19].

Again we introduce the rescaled variables X_μ, and similarly we introduce Φ_α by $\Phi_\alpha = \beta^{3/8}\, \Psi_\alpha$, so that the fermionic action takes the canonical form

$$S^{(\mathrm{F})} = -\frac{i}{2} N \,\mathrm{tr}\Big(\Phi_\alpha (\widetilde{\Gamma}_\mu)_{\alpha\beta} [X_\mu, \Phi_\beta]\Big)\,. \tag{3.3}$$

We write down the SU(N) symmetric gaussian action without assuming SO(10) invariance as

$$S_0 = S_0^{(\mathrm{B})} + S_0^{(\mathrm{F})}\,, \tag{3.4}$$

where $S_0^{(\mathrm{B})}$ is the anisotropic bosonic gaussian action (2.4) and $S_0^{(\mathrm{F})}$ is written as

$$S_0^{(\mathrm{F})} = \frac{N}{2} \sum_{a=1}^{N^2-1} \Phi_\alpha^a\, \mathcal{A}_{\alpha\beta}\, \Phi_\beta^a\,, \tag{3.5}$$

where $\mathcal{A}$ is a 16×16 anti-symmetric matrix.

As before, we introduce Z_0, $\langle\,\cdot\,\rangle_0$ and the free energy is expanded as (2.7). Note that correlation functions $\langle\,\cdot\,\rangle_{\mathrm{C},0}$ including odd powers of $S^{(\mathrm{F})}$ vanish trivially. When we evaluate the free energy F by using the formula (2.7), we have to reorganize the expansion in such a way that we can take into account the three-point interaction $S^{(\mathrm{F})}$ properly. As in ref. [11], we consider $S^{(\mathrm{B})}$, $S_0^{(\mathrm{B})}$, $S_0^{(\mathrm{F})}$, $(S^{(\mathrm{F})})^2$ to be of the same 'order' and arrive at the new expansion

$$F = \sum_{k=0}^{\infty} \tilde{F}_k\,; \qquad \tilde{F}_0 = -\ln Z_0\,; \tag{3.6}$$

$$\begin{aligned} \tilde{F}_k &= -\sum_{l=0}^{k} a_{k,l} \Big\langle (S^{(\mathrm{B})} - S_0)^{k-l} (S^{(\mathrm{F})})^{2l} \Big\rangle_{\mathrm{C},0}\,, \\ a_{k,l} &= \frac{(-1)^{k+l}}{(k+l)!}\; {}_{k+l}\mathrm{C}_{k-l} \qquad (\text{for } k \geq 1)\,. \end{aligned} \tag{3.7}$$

This prescription was first proposed in ref. [12] for general supersymmetric models.[1]

[1] In ref. [12] the gaussian action is constructed using a superfield formalism. Similarly we may use the four-dimensional superfield formalism regarding the IIB matrix model as the zero-volume limit of 4d $\mathcal{N} = 4$ super Yang-Mills theory. The result turns out to be quite similar to what we obtain below for the solution assuming the SO(4) symmetry.

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The zeroth order free energy can be obtained as

$$\tilde{F}_0 = (N^2-1)\left\{ C - \frac{1}{2}\sum_{\mu=1}^{10}\ln v_\mu - \ln(\mathrm{Pf}\,\mathcal{A})\right\}, \tag{3.8}$$

$$C \equiv \frac{1}{2}\left\{ 10\ln(N\beta^{1/2}) - 16\ln(N\beta^{3/4})\right\}. \tag{3.9}$$

The first order correction to the free energy reads

$$\tilde{F}_1 = \langle S^{(\mathrm{B})}\rangle_0 - \langle S_0\rangle_0 - \frac{1}{2}\left\langle (S^{(\mathrm{F})})^2\right\rangle_{\mathrm{C},0}. \tag{3.10}$$

The first term is the same as (2.10) in the bosonic case. The other terms are given as

$$\langle S_0\rangle_0 = -3(N^2-1)\,, \tag{3.11}$$

$$\langle (S^{(\mathrm{F})})^2\rangle_{\mathrm{C},0} = \frac{1}{2}(N^2-1)Q\,, \tag{3.12}$$

$$Q \equiv \sum_\mu \rho_\mu v_\mu\,, \qquad \rho_\mu \equiv \frac{1}{4}\,\mathrm{Tr}\left\{(\mathcal{A}^{-1}\tilde{\Gamma}_\mu)^2\right\}, \tag{3.13}$$

where the trace Tr is taken with respect to the 16-dimensional spinor indices. Thus, at the first order, the free energy is calculated as

$$\frac{1}{N^2-1}(\tilde{F}_0+\tilde{F}_1) = C - \frac{1}{2}\sum_{\mu=1}^{10}\ln v_\mu - \ln(\mathrm{Pf}\,\mathcal{A}) + + \frac{1}{4}\sum_{\mu\neq\nu} v_\mu v_\nu + 3 - \frac{1}{4}Q\,. \tag{3.14}$$

Let us parametrize the 16×16 anti-symmetric matrix $\mathcal{A}_{\alpha\beta}$ in a SO(10) covariant way as

$$\mathcal{A} = \frac{i}{3!}\sum_{\mu\nu\lambda} w_{\mu\nu\lambda}\,\mathcal{B}_{\mu\nu\lambda}\,; \qquad \mathcal{B}_{\mu\nu\lambda} = \mathcal{C}\,\Gamma_\mu\,\Gamma_\nu^\dagger\,\Gamma_\lambda\,, \tag{3.15}$$

where $w_{\mu\nu\lambda}$ is a complex totally anti-symmetric rank-three tensor. As it becomes clear from this parametrization, the fermionic gaussian action breaks the full SO(10) symmetry for any nonzero $w_{\mu\nu\lambda}$. The self-consistency equations at the first order read

$$0 = -\frac{1}{2v_\mu} + \frac{1}{4}\sum_{\nu\neq\mu} v_\nu - \frac{1}{4}\rho_\mu\,, \tag{3.16}$$

$$0 = -\frac{1}{2}\,\mathrm{Tr}(\mathcal{A}^{-1}\mathcal{B}_{\mu\nu\lambda}) + \frac{1}{8}\sum_\mu v_\mu\,\mathrm{Tr}\left\{(\mathcal{A}^{-1}\tilde{\Gamma}_\mu)^2\mathcal{A}^{-1}\mathcal{B}_{\mu\nu\lambda}\right\}. \tag{3.17}$$

In order to go beyond the leading order approximation, we specify the prescription for systematic higher order calculations as follows. We choose the parameters in the gaussian action in such a way that the truncated free energy $\sum_{k=0}^{n}\tilde{F}_k$ is extremized. (Technically, we search for solutions by Newton's method near the solutions found at lower orders.) This

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will give the n-th order approximation to the free energy. Similarly to the free energy (3.6), the expectation value of an operator $\mathcal{O}$ is calculated using the reorganized series expansion

$$\langle \mathcal{O} \rangle = \langle \mathcal{O} \rangle_0 + \sum_{k=1}^{\infty} \tilde{O}_k \, ; \tag{3.18}$$

$$\tilde{O}_k \equiv \sum_{l=0}^{k} a_{k,l} \left\langle (S^{(\mathrm{B})} - S_0)^{k-l} (S^{(\mathrm{F})})^{2l} \mathcal{O} \right\rangle_{\mathrm{C},0} . \tag{3.19}$$

We truncate the infinite series (3.18) at $k = n$, and evaluate it at the solutions to the n-th order self-consistency equations.

As a test, we applied this prescription to a toy model [10] with the action $S = \frac{1}{4g^2}\phi^4$. Here we can easily proceed up to the 10th order (or even higher) by using Mathematica. We found solutions to the self-consistency equation except for the 2nd and 4th orders. At each order (other than 2 and 4), we calculated the free energy and the expectation value $\langle \phi^2 \rangle$. For the free energy, the discrepancy from the exact result is only 0.1% already at order 3, and it becomes 0.004% at order 10. For the expectation value $\langle \phi^2 \rangle$, the discrepancy is found to be 0.4% already at order 3, and it becomes 0.03% at order 10. This suggests that the above prescription for higher order calculations indeed yields a rapid convergence.[2] Details shall be reported elsewhere.

4. Ansatz

Since the gaussian action contains too many parameters (10 real numbers from v_μ in (2.4) and 120 complex numbers from $w_{\mu\nu\lambda}$ in (3.15)), it seems a formidable task to explore the whole solution space of the self-consistency equations. Here we search for solutions assuming that SO(d) plus some discrete subgroup of SO(10) is preserved. For each case below ($d = 2, 4, 6, 7$) the parameters are reduced to two real and one complex numbers. For other values of d, we need to keep more independent parameters, and we leave these cases for future investigations.

First let us assume that SO(7) symmetry is preserved, which allows us to set $v_1 = \cdots = v_7 \equiv V$ and all the $w_{\mu\nu\lambda}$ except $w_{8,9,10}$ to zero. By further imposing the symmetry under a particular SO(10) transformation

$$x_\mu \mapsto x_\nu \, ; \qquad x_\nu \mapsto x_\lambda \, ; \qquad x_\lambda \mapsto x_\mu \, , \tag{4.1}$$

where $(\mu, \nu, \lambda) = (8, 9, 10)$, we set $v_8 = v_9 = v_{10} \equiv v$.

Secondly let us assume SO(6) symmetry, which allows us to set $v_1 = \cdots = v_6 \equiv V$ and all the $w_{\mu\nu\lambda}$ with indices $1 \sim 6$ to zero. We further impose the symmetry under the transformation (4.1) with $(\mu, \nu, \lambda) = (7, 8, 9), (7, 8, 10), (7, 9, 10), (8, 9, 10)$. This requires $v_7 = v_8 = v_9 = v_{10} \equiv v$ and $w_{8,9,10} = -w_{7,9,10} = w_{7,8,10} = -w_{7,8,9}$.

[2] We have also tried other prescriptions. For instance, when we evaluate the expansions truncated at the n-th order, we may use the solution to the *first* order self-consistency equations. However, the results obtained for the free energy and $\langle \phi^2 \rangle$ both start oscillating at the third order, and the oscillation becomes increasingly violent as we go to higher order.

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Next let us assume SO(4) symmetry, which allows us to set $v_1 = \cdots = v_4 \equiv V$ and all the $w_{\mu\nu\lambda}$ with indices $1 \sim 4$ to zero. We further impose the symmetry under the transformation (4.1) with $(\mu, \nu, \lambda) = (8, 9, 10)$ and under a particular SO(10) transformation

$$x_\mu \mapsto x_\nu\,; \qquad x_\nu \mapsto x_\mu\,; \qquad x_1 \mapsto -x_1\,, \tag{4.2}$$

where $(\mu, \nu) = (5, 8), (6, 9), (7, 10)$.

Finally let us assume SO(2) symmetry, which allows us to set $v_1 = v_2 \equiv V$ and all the $w_{\mu\nu\lambda}$ with indices 1 or 2 to zero. We further impose the symmetry under the transformation (4.1) with $(\mu, \nu, \lambda) = (7, 8, 9), (7, 8, 10), (7, 9, 10), (8, 9, 10)$ and under the transformation (4.2) with $(\mu, \nu) = (3, 7), (4, 8), (5, 9), (6, 10)$.

5. Results

For each ansatz preserving the SO(d) symmetry ($d = 2, 4, 6, 7$), we obtain one or two solutions. In the latter case, we only show results for the one which gives the smaller free energy. An analytic formula for the free energy at arbitrary N was obtained in ref. [15] based on explicit numerical evaluations at small N combined with other analytical calculations [20]. At large N, the formula gives[3]

$$\frac{F}{N^2 - 1} = \ln(\sqrt{N} g^7) + \left(\ln 8 - \frac{3}{4}\right) + O\left(\frac{\ln N}{N^2}\right). \tag{5.1}$$

The gaussian expansion reproduces the first term correctly for any solution. Therefore, we compute the 'free energy density' defined by

$$f = \lim_{N \to \infty} \left\{ \frac{1}{N^2 - 1} F - \ln\left(\sqrt{N}\, g^7\right) \right\}. \tag{5.2}$$

The results are shown in table 1. At the first order, the free energy becomes larger for smaller d. At the second order, we find no solutions to the self-consistency equations. This is not so surprising, however, since we encounter a similar situation with the ϕ^4 toy model as we mentioned below (3.19). At the third order, we find that the free energy becomes minimum at the solution preserving SO(4) symmetry. Note also that the value

d	f (order 1)	f (order 3)	ρ (order 1)	ρ (order 3)
2	6.49428	6.50906	2.17736	1.49056
4	6.15335	0.74111	1.85728	3.37766
6	5.75743	1.54414	1.87034	2.24911
7	5.52272	1.62094	1.95533	2.15681

Table 1: The free energy density (f) and the ratio ($\rho = R/r$) of the extents at the order 1 and 3 are shown for each SO(d) preserving solution.

[3] The famous factor [20] $\sum_{n|N} 1/n^2$ in the partition function gives an $O(N^{-2})$ contribution in eq. (5.1), hence it is irrelevant in the present analysis.

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of f obtained for $d = 4$ comes much closer to the 'exact' result ($\ln 8 - 3/4 = 1.32944$) as we proceed from order 1 to order 3. We also calculate the extent in the μ-th direction $R_\mu \equiv \sqrt{\langle \frac{1}{N}\,\mathrm{tr}(A_\mu)^2\rangle}$. Note that $R_1 = \cdots = R_d \equiv R$ and $R_{d+1} = \cdots = R_{10} \equiv r$ due to the imposed symmetry. At the first order, the ratio $\rho \equiv R/r$ is given by $\sqrt{V/v}$ and we find that $\rho > 1$. At the third order, we observe that the ratio ρ increases in all the cases except for $d = 2$.

6. Discussion

In this paper we have formulated an analytical approach to the spontaneous breakdown of SO(10) symmetry in the IIB matrix model. Our approach is based on the gaussian expansion technique, which was quite successful in the Matrix Theory even at the leading order. We have given a prescription for systematic higher order calculations, which, to our surprise, yields a rapid convergence in a simple example.

Here we have carried out our program up to the third order, which can be done with reasonable efforts (e.g., we evaluated 34 four-loop diagrams). We found various solutions which break SO(10) symmetry, and among them the SO(4) preserving solution turned out to give the smallest free energy. Moreover the ratio of the extents in the 4 and the other 6 directions increases as we go to higher order. These results support the conjectured scenario that 4d space-time is generated dynamically in nonperturbative superstring theory. Let us also recall that the emergence of four-dimensional space-time in the IIB matrix model has already been suggested in ref. [8] based on Monte Carlo results and the branched polymer description [2] of its low-energy effective theory. It is encouraging that we arrive at '4d' from a totally different approach.

The analytic formula for the free energy obtained by Krauth-Nicolai-Staudacher [15] provides a useful guide-line for the convergence of the present approach. Indeed we observed that the free energy calculated for the SO(4) preserving solution comes much closer to the KNS result as the order is increased. In the bosonic model, the gaussian expansion (even at the first order) becomes *exact* in the large-D limit [10]. This fact can be understood naturally from the viewpoint of a systematic $1/D$ expansion [3]. In the supersymmetric models, D is restricted to 4, 6 and 10 and therefore 'the large-D limit' does not make sense. Still it is conceivable that the convergence is reasonably fast if D is as large as 10, i.e. the case corresponding to the IIB matrix model. The same reasoning explains the success of the (leading-order) gaussian approximation in the Matrix Theory [14]. Nevertheless it is certainly worth while to perform higher order calculations and confirm the convergence directly. Incidentally we had to restrict the parameter space of the gaussian action for a purely technical reason. It would be interesting to enlarge the parameter space to study other SO(d) preserving solutions, in particular $d = 3$ and $d = 5$.

Finally we would like to comment on the scale parameter g in the action (2.2) and (3.2). In the present approach, we rescale the matrices in such a way that the action takes the canonical form (2.3), (3.3). This corresponds to setting $g^2 N = 1$ in the original action. Then the N dependence is eliminated from the self-consistency equations. If $\langle \mathcal{O} \rangle$ converges in high order calculations, it means that the quantity becomes finite in the large-N limit with g^2N fixed. This explains the observed large-N behavior of the space-time extent and the Wilson loops in the bosonic and supersymmetric models [3, 18]. In the IIB matrix model, it is therefore plausible that g^2N should be fixed in order to obtain finite Wilson loop correlators in the large-N limit. On the other hand, Monte Carlo results [8] suggest that the extent of the space-time (in the 4 directions) may diverge in this large-N limit. It would be interesting to see if such trends appear in the higher order calculations. We hope that our analytical approach to the IIB matrix model is useful to extract more information on its highly nontrivial dynamics.

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